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HOMOGENEITY OF DEGREE IN COMPLEX SOCIAL NETWORKS AS A COLLECTIVE GOOD

Gregory Todd Jones,* Douglas H. Yarn,** Reidar Hagtvedt,*** & Travis Lloyd****

INTRODUCTION

Cooperation has played a prominent role in the evolution of many species, from the simplest single-celled organisms 1 to fish, 2 from birds 3 to canines 4 and felines 5 and from non-human primates 6 to 7.
humans, where cooperation may have had the most evolutionary significance. And yet, the evolution of cooperation among self-regarding individuals remains a formidable challenge currently addressed by highly multi-disciplinary efforts that include scientists from anthropology, biology, computer science, ecology, economics, physics, political science, psychology, mathematics, sociology and numerous other fields. The puzzles posed by cooperative behavior take many forms, but at their root, all involve social dilemmas – circumstances in which individual interests are at odds with common interests. More precisely, individuals are faced with a choice between selfish behavior and prosocial, cooperative behavior where the latter imposes more cost or offers less benefit than the former. While all individuals are strictly better off being selfish, regardless of what other individuals choose to do, all individuals would be best off if enough individuals behaved cooperatively. Thus, the dilemma. The study of these types of problems has largely been driven by the


7. See generally Ernst Fehr & Urs Fischbacher, The Nature of Human Altruism, 425 NATURE 785 (2003); Dominic Johnson et al., The Puzzle of Human Cooperation, 421 NATURE 911 (2003); Elinor Ostrom et al., Revisiting the Commons: Local Lessons, Global Challenges, 284 SCIENCE 278 (1999).


9. Id.

10. See generally ROBYN M. DAWES & DAVID M. MESSICK, SOCIAL DILEMMAS, 35 INT'L J. PSYCHIATRY 111 (2000); N. M. Gotts et al., Agent-Based Simulation in the Study of Social Dilemmas, 19 ARTIFICIAL INTELLIGENCE REV. 3 (2003).
application of evolutionary game theory\textsuperscript{11} and due to their importance as generalized models of many important socio-economic situations,\textsuperscript{12} iconic games such as the Prisoner's Dilemma have been widely employed as metaphors for the dilemma.\textsuperscript{13}

At the same time, the study of networks, complex systems, and nonlinear dynamics has pervaded all of science,\textsuperscript{14} offering insight into such diverse concerns as the architecture of the Internet,\textsuperscript{15} the topology of food webs,\textsuperscript{16} and the metabolic network of the bacterium \textit{Escherichia coli}.\textsuperscript{17} Indeed, E.O. Wilson, who once characterized the evolution of cooperation as one of the greatest challenges for modern biology,\textsuperscript{18} more recently made a more emphatic appeal for research on complex systems.\textsuperscript{19}

The greatest challenge today, not just in cell biology and ecology, but in all of science, is the accurate and complete description of complex systems. Scientists have broken down

\begin{footnotesize}


\textsuperscript{17} H. Jeong et al., \textit{The Large-Scale Organization of Metabolic Networks}, 407 NATURE 651 (2000).


\textsuperscript{19} E. O. WILSON, CONSILIENCE 85 (1998).
\end{footnotesize}
many kinds of systems. They think they know most of the elements and forces. The next task is to reassemble them, at least in mathematical models that capture the key properties of the entire ensembles. 20

The application of complex systems tools and network analysis methodologies to the study of social dilemmas represents a very new, but extremely promising means of shedding light on the quandary of cooperation. 21

Early simulation studies of the evolution of cooperation were based on the notion that all agents competed with all other agents, including themselves, in a round robin style tournament that pitted various interaction strategies against all others. 22 Later studies demonstrated that when evolutionary dynamics were at play, the evolution of cooperation depended on what philosopher Bryan Skyrms called "correlated association," 23 that is, that it was at least slightly more likely that agents would interact with a subpopulation of agents of their own kind, in strategic terms. This correlated association can be accomplished in many ways, including reputational mechanisms, signaling mechanisms, and spatiality.

20. Id.

22. Known as a "mean field" simulation. Axelrod’s early tournaments were archetypal. See Axelrod, Evolution of Cooperation, supra note 13.
Following the common sense notion that geography might play a role in the evolution of cooperation, many simulation studies have employed spatiality, placing agents on a two-dimensional grid. 24 (See Figure One.) Side effects of this typology include an artificial limitation on the number of neighbors, or social connections (known as “degree” in network analysis parlance) and a concurrent limitation on the possible differences in levels of connections between individual agents (known as “heterogeneity of degree”).

In this study, we join those that employ complex systems tools and network analysis methodologies 25 to leave the artificiality of the two-dimensional toroidal architecture behind in favor of network architectures offering a full range of degree and heterogeneity of degree, facilitating a more generalized study of the evolution of prosocial behavior. (See Figure Two.) In what follows, we begin by formally specifying the models under study and providing a detailed description of the simulations. We then explain our results, focusing principally on the conclusion that heterogeneity of degree negatively influences the evolution of cooperation and that this effect is independent from other factors such as average degree.

24. Where this two-dimensional grid is “non-bordered,” that is, the top wraps to the bottom and the left side wraps to the right side, the typology is called a torus.

25. See supra note 21.
Finally, we briefly discuss the consequences of these results in the broader context of institutional design efforts that may bring about increased levels of cooperation and maximize social welfare. We suggest that the promotion of homogeneity of degree may properly be viewed as a collective good.
MODELS AND SIMULATIONS

We examine a population of $N$ players, each engaging in a repeated prisoner's dilemma game with a neighborhood of other players defined by particular network architectures. The set of players with whom player $i$ interacts in period $t$ is denoted by $\Omega_i$. In each generation, which is comprised of $g$ games, each player accumulates an adaptive score based upon a standard payoff matrix described in more detail below. At the end of each generation, each player

26. For an introduction to the significance of network architecture, see Xiao Fan Wang & Guanrong Chen, Complex Networks: Small-World, Scale-Free and Beyond, IEEE CIRCUITS AND SYSTEMS MAGAZINE 6 (2003). See Figure Three.
observes the payoffs and strategies of each neighbor and stochastically updates\(^\text{27}\) their strategy with probability \(p \in [0,1]\) by imitating the strategy of the neighbor with the highest adaptive score (including themselves). Ties in high scores are broken at random.

![Figure Three: Schematic illustration of various network architectures, all with 25 nodes, roughly in ascending order of heterogeneity. (a) Fully connected network. (b) Ring lattice with all nodes connected to its neighbors out to some range \(k\) (here \(k = 3\)). (c) Small world network starting with ring lattice and adding shortcut links between random pairs of nodes. (d) Random network constructed with connection probability, \(p = .15\). (e) Scale-free network constructed by attaching nodes at random to previously existing nodes, where the probability of attachment is proportional to the degree of the target node, i.e., "the rich get richer." (full color diagram available at: http://www.gregorytoddjones.com/publications.htm).]

**Strategic Dynamics**

For each period \(t\), players choose to either cooperate (\(C\)) or defect (\(D\)) with each of its neighbors \(\Omega_{i,t}\) and the strategic decision for each neighbor is independent of the decisions with regard to other

neighbors, that is, a player can choose to cooperate with some neighbors and defect with others. Each neighbor $j \in \Omega_{i,t}$ faces a symmetrical decision giving rise to a standard payoff matrix.

<table>
<thead>
<tr>
<th>$i,j$</th>
<th>$C$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$R, R$</td>
<td>$S, T$</td>
</tr>
<tr>
<td>$D$</td>
<td>$T, S$</td>
<td>$P, P$</td>
</tr>
</tbody>
</table>

Where $\pi(s_i, s_j)$ is the payoff for player $i$ choosing strategy $s_i$ when neighbor $j$ chooses strategy $s_j$,

$$\pi(C, C) = R, \quad \pi(C, D) = S, \quad \pi(D, C) = T, \quad \text{and} \quad \pi(D, D) = P.$$ 

In keeping with the standard structure of a social dilemma, $T > R > P > S$, which makes defection a dominant strategy, that is, defection results in a higher payoff as compared to cooperation, regardless of what strategy the opponent neighbor chooses, and $2R > (T + S)$, which insures that mutual cooperation is preferred over all other strategy sets in the sense that it produces maximum aggregate outcomes. The unique equilibrium for the game, mutual defection, thus leads to a Pareto-suboptimal solution.

For each generation, each player accumulates an adaptive score for $g$ games for all neighbors. Following the logic that the maintenance of networks with more neighbors would involve more cost than networks with fewer neighbors, we reduce adaptive scores by $\theta(k)$, the total cost of interaction with a network of $k$ neighbors. Thus, the net payoff for each player $i$ accumulated in a time period $t$ is

$$\Pi_{i,t} = \sum_{j \in \Omega_{i,t}} \pi(s_{i,t}, s_{j,t}) - \theta(k_{i,t}),$$

where $\theta(k)$ is an increasing function of $k$ with the specific form $\theta(k) = ck^a$, where $\alpha \geq 1$ and $0 \leq c \leq P$. 29

28. But see Hanaki, supra note 21.
29. See Hanaki, supra note 21.
**Imitation Dynamic**

After each generation, each player examines the accumulated adaptive scores of each of its neighbors, $\Omega_{i,t}$, and its own accumulated adaptive score, and either adopts by imitation the strategy of the most successful neighbor, or keeps its own strategy if it has been most successful, to be employed in the next generation, formally

$$s_{i,t+1} = \arg\max_{j \in \Omega_{i,t}} \Pi_{j,t}(s_{j,t}) \Pi_{i,t}(s_{i,t}).$$

If more than one player in the neighborhood shares the highest accumulated adaptive score, ties are broken at random.

**Network measures**

For each run of the simulation, which is comprised of a large number of generations sufficient to arrive at equilibrium in the strategy population, a number of variables are recorded: population, average degree, heterogeneity of degree, network architecture, and cooperation. Network architecture is recorded as lattice, small world, random, or scale-free (fully connected is a special case of lattice). Cooperation is measured as a ratio of player decisions to cooperate to the total number of cooperation/defection decisions. (See Table One.)

**RESULTS**

We ran the simulation as described above 1,000 times creating stochastic networks by drawing network architecture uniformly from lattice, small world, random, or scale-free; drawing population uniformly from a range of 10 to 100; and drawing average degree uniformly from a range of 2 to 10. Heterogeneity of degree ranged from 0 to 4 as a function largely of network architecture. Each run was for 1,000 generations with the cooperation ratio measured in the last 100.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Variable Type</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population: the number of players – constant throughout a given simulation run</td>
<td>Independent</td>
<td>$N$</td>
</tr>
<tr>
<td>Average Degree: the average number of social network connections across all players</td>
<td>Independent</td>
<td>$\bar{k} = \frac{1}{N} \sum_{i} k_i$</td>
</tr>
<tr>
<td>Heterogeneity: the standard deviation of average degree across all players</td>
<td>Independent</td>
<td>$\sigma = \sqrt{\sigma^2}$ where $\sigma^2 = \sum_{k=1}^{N-1} (k - \mu)^2 \cdot d(k)$ and $\mu = \sum_{k=1}^{N-1} k \cdot d(k)$</td>
</tr>
<tr>
<td>Network Architecture</td>
<td>Independent</td>
<td>Indicator Variables</td>
</tr>
<tr>
<td>Cooperation</td>
<td>Dependent</td>
<td>The ratio of acts of cooperation to opportunities for cooperation.</td>
</tr>
</tbody>
</table>

**Table One: Model Variable Specification**

First, we regressed cooperation on population, average degree, heterogeneity of degree, and three indicator variables representing four network architectures: lattice (as the base case), small world, random, and scale free. (See Model One.) We included the indicator variables to capture any variation resulting from network architectural differences not captured by the other independent variables.
Model One: Cooperation ("Coop") regressed on Population ("Pop"), Average Degree ("Degree"), Heterogeneity of Degree ("Hetero") and three indicator variables representing four network architectures, Lattice (base case), Small World ("SWDum"), Random ("RNDum"), and Scale Free ("SFDum").

We hypothesized that population size would have a positive effect on cooperation, and that both average degree and heterogeneity of degree would have a negative effect. While Model One bore out the first two hypotheses (population coefficient = .245, p < .000 and average degree coefficient = -8.170, p < .000), heterogeneity of degree showed a significant positive effect (heterogeneity of degree coefficient = 31.847, p < .000). However, collinearity diagnostics indicated that the two most heterogeneous network architectures were highly collinear with the heterogeneity of degree variable (variance proportions on dimension 7: random network = .91, scale-free network = .94, and heterogeneity of degree = .91). Further, the small-
world network indicator variable failed to achieve statistical significance ($p = .660$). These results offered confidence that the indicator variables were not adding significantly additional explanatory power. Indeed, the collinearity made model coefficients uninterpretable.

### Coefficients

<table>
<thead>
<tr>
<th>Model</th>
<th>Unstandardized Coefficients</th>
<th>Standardized Coefficients</th>
<th>t</th>
<th>Sig</th>
<th>Collinearity Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(Constant)</td>
<td>43.468</td>
<td>2.372</td>
<td>18.328</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Pop</td>
<td>.335</td>
<td>.035</td>
<td>.428</td>
<td>9.473</td>
</tr>
<tr>
<td></td>
<td>Degree</td>
<td>-4.780</td>
<td>.279</td>
<td>-.772</td>
<td>-17.146</td>
</tr>
<tr>
<td></td>
<td>Hetero</td>
<td>-4.208</td>
<td>1.204</td>
<td>-.154</td>
<td>-3.495</td>
</tr>
</tbody>
</table>

a. Dependent Variable: Coop

### Collinearity Diagnostics

<table>
<thead>
<tr>
<th>Model</th>
<th>Vector</th>
<th>Condition Index</th>
<th>Variances Proportions</th>
<th>Variance Proportions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Constant)</td>
<td>1.000</td>
<td>.01</td>
<td>.02</td>
</tr>
<tr>
<td></td>
<td>Pop</td>
<td>2.750</td>
<td>.01</td>
<td>.02</td>
</tr>
<tr>
<td></td>
<td>Degree</td>
<td>3.843</td>
<td>.09</td>
<td>.21</td>
</tr>
<tr>
<td></td>
<td>Hetero</td>
<td>5.580</td>
<td>.88</td>
<td>.75</td>
</tr>
</tbody>
</table>

a. Dependent Variable: Coop

Model Two: Cooperation ("Coop") regressed on Population ("Pop"), Average Degree ("Degree"), and Heterogeneity of Degree ("Hetero")

Subsequently, in Model Two, we removed the indicator variables and regressed cooperation on population, average degree, and heterogeneity of degree. (See Model Two.) In this more parsimonious model, population had a significant positive effect (population coefficient = .428, $p < .000$), average degree had a significant negative effect (average degree coefficient = -4.78, $p < .000$), and heterogeneity of degree had a significant negative effect (heterogeneity of degree coefficient = -4.208, $p < .001$). Additionally, collinearity diagnostics showed that each of the independent variables was loading highly on its own dimension (population variance proportion on dimension 4 = .75, average degree
variance proportion on dimension 3 = .86, heterogeneity of degree variance proportion on dimension 2 = .93). Based on this evidence, we concluded that heterogeneity of degree has a significant negative effect on the evolution of cooperation and that this effect is independent of the negative effect of average degree.

**DISCUSSION**

In *Bowling Alone*, Robert Putnam worries that the decline of social capital that he sees in the declining memberships in civic organizations may undermine the civil engagement that, according to him, is necessary for a strong democracy. The results of this study suggest that the problem may be more nuanced. It may not be, in fact, the mere magnitude of social connections but the nature of these connections that should concern us most. As Putnam points out, membership in local civic organizations has been replaced to some extent by mass membership organizations, and our results demonstrate that resulting increases in average degree may exert a negative influence on cooperative behavior that promotes social welfare. Putnam's work also suggests that local cohesiveness, or "clumpiness" may have a determinative effect. This is a network measure not included here, but planned for future studies.

Our most important finding in this study, however, is that inequality in social connectedness, heterogeneity of degree, has a negative effect on the evolution of prosocial behavior, and that this effect is independent of the negative effect of average degree. Paired with evidence that modern day social and technological networks are increasing in heterogeneity, exhibiting multi-peaked degree distributions unlike the egalitarian, single-peak degree

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distributions\textsuperscript{32} characteristic of the Pleistocene's environment of evolutionary adaptedness (EEA)\textsuperscript{33} when our current social brains evolved, these findings should give us pause. Merely promoting the development of dense social networks may lead us down a path to social decline. More important may be the design of institutions that promote homogeneity in social connectedness – increasing homogeneity of degree produces network effects that increase overall social welfare. As such, homogeneity of degree is properly thought of as a collective good.


\textsuperscript{33} John Bowlby, \textit{Attachment} (1969).