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# **RANDOMNESS AND COMPLEXITY IN SOCIAL EXPLANATION: EVIDENCE FROM FINANCE AND BANKRUPTCY LAW**

**Bernard Trujillo\***

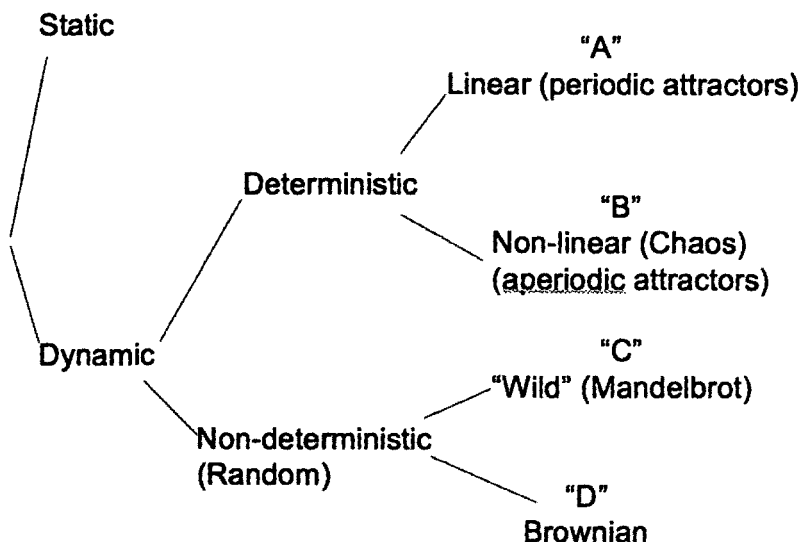
Quantitative models are useful tools for understanding and explaining both natural and social systems. Models often include a term representing a random or stochastic element. Random terms are commonly deployed in modeling social phenomena such as economic, financial, and legal systems. This article contrasts conventional random terms in quantitative models with alternative terms supplied by the mathematics and science of complexity. This article argues that complexity modeling can explain many of the social phenomena that interest researchers. This article concludes with preliminary applications of complexity modeling in finance and bankruptcy law.

## **INTRODUCTION: WHAT DYNAMICS EXPLAIN SOCIAL FORMS?**

We want to understand the dynamics that generate the things we observe. What are the rules, equations, interactions, or forces that produce objects and events in the world? A meteorologist wants to understand the forces that yield a storm or a still night. A financial scientist wants to understand the influences behind the daily movement of stock prices. And a student of legal systems wants to understand the forces that explain the diffusion of doctrine across space, or the rise and fall of legal forms throughout time.

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**Figure 1: Taxonomy<sup>1</sup>**

Figure 1 is a rough taxonomy of the sorts of dynamics that produce the things we see in the world.<sup>2</sup> The initial division is between “static” systems, which do not change over time, and “dynamic” systems, which do.

The category of dynamic systems divides into “deterministic” systems and “non-deterministic” systems. Deterministic systems behave according to a specified set of rules or equations that determine the next state of the system based on the current state of the system. Suppose your rule is always to turn on your front-porch light only when both of your immediate neighbors have turned on their front-porch lights, and to turn your light off only when both of your neighbors have turned off theirs. If I know the rule and the

1. This is my own diagram, but leans on Strogatz and Sprott, both cited below.

2. The generation of this Figure relies on tables by Strogatz and Sprott. See STEVEN H. STROGATZ, *NONLINEAR DYNAMICS AND CHAOS* 10 (Westview 1994); JULIEN CLINTON SPROTT, *CHAOS AND TIME-SERIES ANALYSIS* 212 (Oxford 2003). The type of “wild” randomness denoted at level “C” is something of an intriguing wildcard, since it does not fit comfortably within the “point-to-point independence” definition of randomness set forth below. See *infra* Part I and note 34.

current state of the lights on your street, I can predict the next state of your light.

Non-deterministic systems, on the other hand, exhibit state-to-state independence. Nothing in the arrangement of the system at time-one will determine the arrangement of the system at time-two. This sort of point-to-point independence is generally what we mean by “randomness.”

Figure 1 lists two types of deterministic dynamics, along with the sorts of forms, or “attractors” that these dynamics produce. Linear deterministic systems (“A” in Figure 1) can be complicated systems of many parts, or they can be very simple systems with just a few parts. But every linear system is essentially modular – one can successfully analyze the system by breaking it down into parts and measuring each part separately. A linear system is no more or less than the sum of its parts.<sup>3</sup> The out-product of linear systems is regular, or periodic.<sup>4</sup>

The other type of deterministic system listed in Figure 1 is “nonlinear.”<sup>5</sup> A nonlinear system (“B” in Figure 1) cannot be analyzed by breaking it into modules. Integral to the system is cooperation among, or competition between, variables making the nonlinear system always more (or less) than the sum of its parts.<sup>6</sup> Characteristic of nonlinear systems is the emergence of new forms or behaviors that were not part of the initial system. Nonlinear systems are capable of generating “aperiodic” attractors, so-called because the trajectory of the attractor never repeats.

It is possible to predict the behavior of nonlinear systems in the very short term, but not much beyond that. Assuming we had perfect

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3. STEVEN STROGATZ, *SYNC: THE EMERGING SCIENCE OF SPONTANEOUS ORDER* 50–51 (Hyperion 2003).

4. Strogatz notes that linear systems are incapable of rich behavior. STROGATZ, *supra* note 3, at 51.

5. A chaotic system is a type of nonlinear deterministic system that exhibits sensitive dependence on initial conditions. SPOTT, *supra* note 2, at 104 (“chaos is the aperiodic, long-term behavior of a bounded, deterministic system that exhibits sensitive dependence on initial conditions”). A common illustration of “chaos” is that a butterfly, flapping its wings in Brazil, can cause tornadoes in Texas. Chaotic systems are necessarily produced by nonlinear rules. It is also possible, however, for nonlinear rules to produce regular, periodic behavior (e.g. planetary motion).

6. STROGATZ, *supra* note 3, at 50–51.

knowledge of the system's governing equations and of all the variables in the system (heroic assumptions, indeed), we would be able, at time-one, to predict the state of the system at time-two. But even assuming heroic knowledge, we would probably be unable, at time-one, to predict the state of the system at time-three. And our ability to predict declines precipitously as the iterations of the system increase. Thus an entirely deterministic system can be (and often is) unpredictable as a practical matter.

Figure 1 also lists two possibilities for non-deterministic systems: "wild" (named as such by the mathematician Benoit Mandelbrot<sup>7</sup> and denoted as "C" in Figure 1) and "Brownian" ("D" in Figure 1). I shall say more about these two types of randomness in Part I of this Article.

We can impose two further axes on Figure 1: predictability and capacity to generate complex structures or forms.

*Predictability.* The systems near the top of Figure 1 ("static" and level "A" linear determinism) are predictable. Level "B" determinism, as we have said, is predictable only under very constrained circumstances. And Level "C" and "D" randomness are, by definition, unpredictable.

*Capacity of the system to generate complex forms.* While linear systems are capable of producing some interesting behavior, most phenomena worth study cannot be generated by linear systems

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7. BENOIT B. MANDELBROT, *FRACTALS AND SCALING IN FINANCE: DISCONTINUITY, CONCENTRATION, RISK* 120 (Springer-Verlag 1997). *See also* Mandelbrot's discussion of Paul Levy's work at BENOIT B. MANDELBROT & RICHARD L. HUDSON, *THE (MIS)BEHAVIOR OF MARKETS: A FRACTAL VIEW OF RISK, RUIN, AND REWARD* 160-61 (2004).

alone.<sup>8</sup> Complex and interesting forms can be generated by nonlinear deterministic systems<sup>9</sup> and by random systems.<sup>10</sup>

Modeling of social phenomena has typically relied on linear mathematics with a stochastic term thrown in. That is, most modeling utilizes the edges of Figure 1 (some level “A” linearity with a dash of level “D” Brownian randomness). This Article intends to draw attention rather to the middle of Figure 1. We can model crucial social forms, like the movement of stock prices or the diffusion of legal doctrine, with nonlinear chaotic systems (level “B”) and with the sort of “wild” randomness (level “C”) possessing a fractal quality that is a signature of complex systems.<sup>11</sup>

## I. RANDOMNESS

“Brownian motion” is one common representation of the point-to-point independence that we have defined as randomness. The phenomenon is named for the Scottish botanist Robert Brown, who studied pollen molecules suspended in water.<sup>12</sup> When viewed under a microscope, Brown saw that the pollen moved in unpredictable ways.<sup>13</sup> Figure 2 illustrates an example of Brownian motion.

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8. Thus mathematician Stanislaw Ulam’s famous observation that the study of non-linear science is like the study of “non-elephant animals.” Most animals are non-elephants, and most phenomena are nonlinear. *See, e.g.,* David K. Campbell, *Nonlinear Physics: Fresh Breather*, 432 NATURE 455, 455–56 (Nov. 25, 2004) (“Stanislaw Ulam, the celebrated Polish mathematician and godfather of the field now known as nonlinear science, famously remarked that using the term ‘non-linear science’ was like ‘calling the bulk of zoology the study of non-elephants’.” He meant that linear processes are the exception rather than the rule; that most phenomena are inherently nonlinear; and that the effects of nonlinearity are apparent everywhere in nature, from the synchronized flashing of fireflies through clear-air turbulence to tornadoes and tsunamis.”).

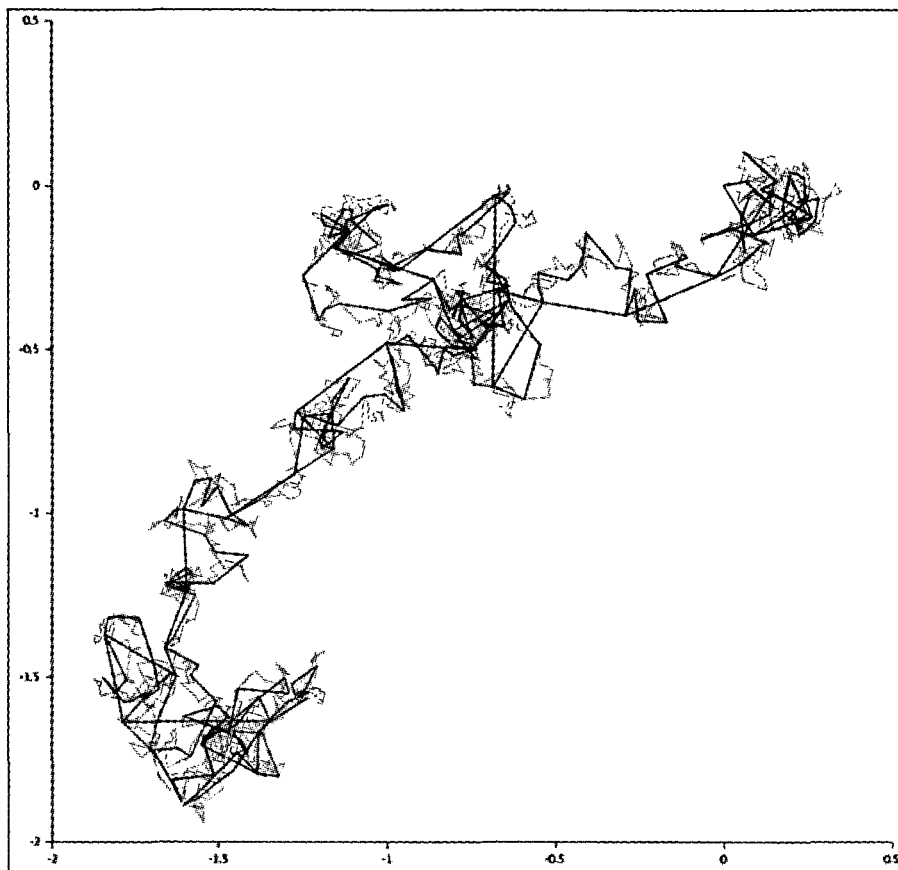
9. *See infra* Part II.

10. *See generally* MANDELBROT, *supra* note 7.

11. *See generally* SPROTT, *supra* note 2 at 273ff (“Fractals are to chaos what geometry is to algebra. They are the usual geometric manifestation of the chaotic dynamics.”). Fractals possess some degree of self-similarity (complete self-similarity if the fractal is generated by deterministic dynamics, and statistical self-similarity if it is a random fractal) such that its visual representation is scale invariant across space.

12. *See* SPROTT, *supra* note 2 at 226, n.16.

13. *See* Robert Brown, *A Brief Account of Microscopical Observations Made in the Months of June, July, and August, 1827, on the Particles Contained in the Pollen of Plants; and on the General Existence of Active Molecules in Organic and Inorganic Bodies*, 4 THE PHILOSOPHICAL MAGAZINE AND ANNALS OF PHILOSOPHY 161–173 (Sept. 1828); *see also* J.L. Doob, *The Brownian Movement and*



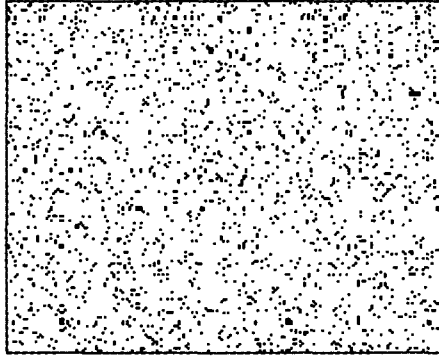
**Figure 2: Example of Brownian Motion<sup>14</sup>**

One characteristic of this type of randomness is a lack of structure or pattern. Figure 3 provides a geometric representation of Brownian randomness.<sup>15</sup>

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*Stochastic Equations*, 43 ANNALS OF MATHEMATICS 351–69 (Jan. 14, 1942). Einstein offered a theory of Brownian motion in one of his “miracle year” papers of 1905. See Albert Einstein, *Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen*, 17 ANNALEN DER PHYSIK 549–60 (1905). See also ALBERT EINSTEIN, INVESTIGATIONS ON THE THEORY OF BROWNIAN MOVEMENT (Dover 1956).

14. Brownian tracks. [http://en.wikipedia.org/wiki/Image:Brownian\\_hierarchical.png](http://en.wikipedia.org/wiki/Image:Brownian_hierarchical.png).



**Figure 3: Randomness geometrically represented<sup>16</sup>**

The eye scans and finds no resting place. Data points fill the space more or less uniformly. The attractor generated by this sort of random dynamics appears geometrically as a sort of smear across space.<sup>17</sup> In nature, random dynamics of this sort produces static (such as the static on the radio or television) and the process of radioactivity.<sup>18</sup>

The randomness term in models can take many forms. One common way of utilizing randomness is through a “Markov process” that utilizes the flip of a fair coin to move about a grid.<sup>19</sup> For example, suppose standing on a street corner in a grid-like urban area. You flip a coin once (say, heads means you go North/South and

15. Figure 3 is an iterated function system using white noise (random data). See SPROTT, *supra* note 2, at 353.

16. Iterated function system with random data (white noise). <http://sprott.physics.wisc.edu/phys505/lect14.htm>.

17. Brownian randomness is the sort commonly utilized in modeling, but it is not the only form of randomness. Mandelbrot, following Lévy, offers a list of seven forms of randomness. See MANDELBROT, *supra* note 7, at 140–41; see generally PAUL LÉVY, *PROCESSUS STOCHASTIQUES ET MOUVEMENT BROWNIEN*, (Gauthier-Villars 1965). Mandelbrot utilizes another form of randomness he names “wild” to generate his models of stock price movements. See *infra* Part IV.

18. See SPROTT, *supra* note 2, at 212.

19. See generally BHARUCHA-REID, A. T. *ELEMENTS OF THE THEORY OF MARKOV PROCESSES AND THEIR APPLICATIONS* (McGraw-Hill 1960); ATHANASIOS PAPOULIS, *BROWNIAN MOVEMENT AND MARKOFF PROCESSES* 515–53 (McGraw-Hill 1965). In a Markov process, the probability of the next state is limited by the present state, thus resulting in a relatively smooth distribution. Notably, a Markov process is not entirely consonant with the “point-to-point independence” definition of randomness – the next state of a Markov process is dependent on the present state. Another random term is the “Martingale,” a mechanism developed by mathematician Paul Levy. See also WILLIAM FELLER, *AN INTRODUCTION TO PROBABILITY THEORY AND ITS APPLICATIONS* 210–15 (Wiley 1971).



tails means you go East/West) and the coin comes up heads. Then you flip again (say heads means North and tails means South) and it comes up heads. You walk one block North and flip again. This time the two coin-flips tell you to walk one block West. And so forth. This is a form of “Markov process” that is sometimes referred to as a “drunkard’s walk.”

Random terms such as a Markov process are part of the equipment of important models of social phenomena, including the “Efficient Capital Markets Hypothesis,” (“ECMH”) which has been the subject of much research in both finance and law.<sup>20</sup> The ECMH explains daily changes in stock prices as a random system exhibiting point-to-point independence, such that price movements are explained by the introduction of new information and the quick absorption of that information into price.<sup>21</sup>

Brownian randomness has been an important explanatory tool. Our reliance on random terms might be partly explained by the limitations of our calculation technology. Modelers trying to explain some complex phenomenon often “put the rabbit in the hat” by including the standard stochastic term, which in turn generates a surprising form. The presumption had been that we needed to build unpredictability into the equation in order to generate the sort of forms that we see in the world. For decades the standard stochastic terms have performed well in the important job of getting our models up and running. As our computing power and knowledge has advanced, however, we are in a position to ask whether reliance on

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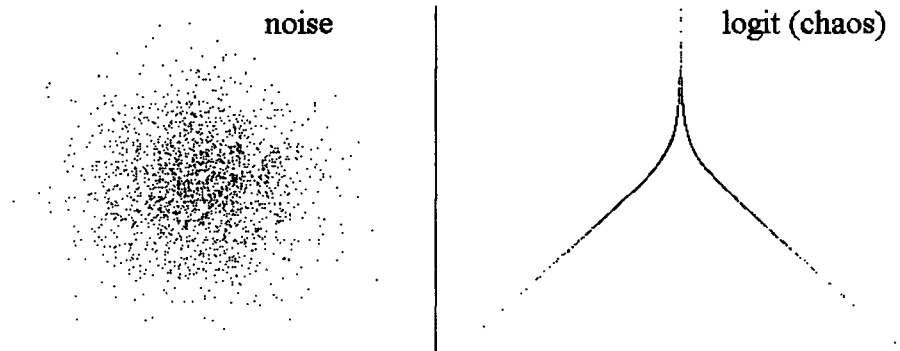
20. See Eugene F. Fama, *Efficient Capital Markets: A Review of Theory and Empirical Work*, 25 THE JOURNAL OF FINANCE 383 (1970); Ronald J. Gilson & Reinier H. Kraakman, *The Mechanisms of Market Efficiency*, 70 VA. L. REV. 549 (1984); Donald C. Langevoort, *Foreword: Revisiting Gilson and Kraakman's Efficiency Story*, 28 J. CORP. L. 499 (2003).

21. The ECMH states a system of Brownian randomness plus exogenous shocks. The ECMH claims, in short, that the stock market will produce gently clustered movements except when it does not. As a bit of mathematics, the ECMH starts with the phenomenon it is trying to explain (i.e. the movement of stock prices) and jerry-rigs an explanation to replicate the phenomenon. The ECMH is thus an example of a hypothesis existing entirely within the phenomenon it is designed to explain. There is reason to be suspicious of an explicans that is limited to, and works backwards from, the explicandum. One thinks of Ptolemy's epicycles laboriously saving the phenomenon of a geocentric solar system. See generally, Michael H. Shank, *Regiomontanus on Ptolemy, Physical Orbs, and Astronomical Fictionalism*, 10 PERSPECTIVES ON SCIENCE 2, 179–207 (2002).

Brownian randomness in modeling is simply a cloak for our ignorance.<sup>22</sup> Perhaps other terms might also generate complex and interesting forms like those we see in the world.

## II. COMPLEXITY ALTERNATIVE

We have seen that standard random terms in models can generate complex forms. May deterministic equations also generate solutions that look like complex forms? We start with a simple side-by-side comparison of a random system with a very simple deterministic system in Figure 4.



**Figure 4: Return map for Random and Chaotic data  
(fuzzy ball v. structure)<sup>23</sup>**

The left side shows a return map (i.e. a plot showing each value of a time series as a function of its previous values) of random data, with the right side showing a return map of data for a very simple deterministic system where each value depends only on the value of its immediate predecessor.<sup>24</sup> Where the return map of random data

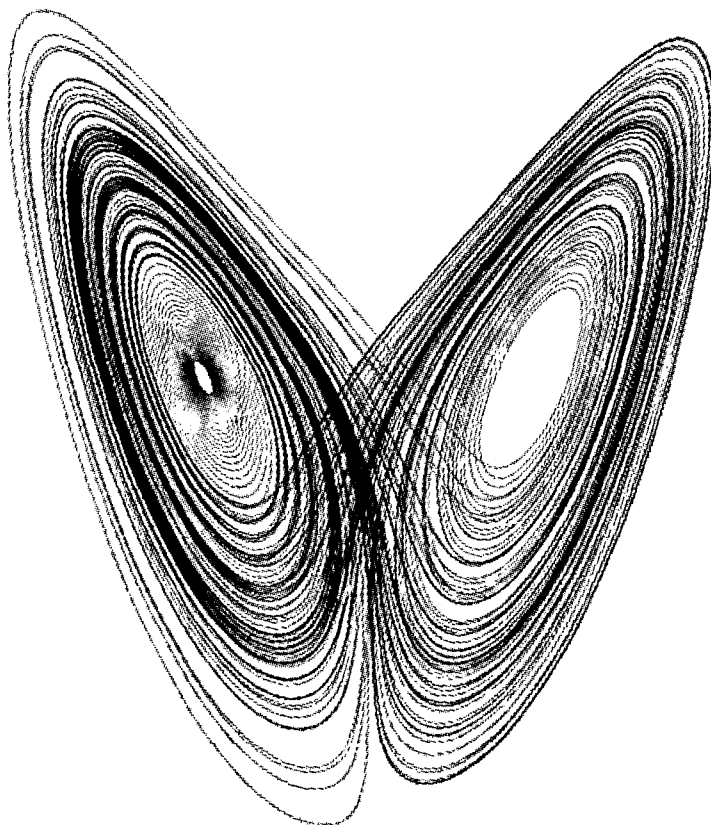
22. "The random assumption is a way of throwing up one's hands, a null hypothesis in the absence of any information." STROGATZ, *supra* note 2, at 237.

23. See SPOTT, *supra* note 2, at 235.

24. See SPOTT, *supra* note 2, at 235–236. The right side shows a return map of  $X_{n+1}$  versus  $X_n$  for a deterministic system where  $X_n$  depends on the value of its immediate predecessor.

shows a fuzzy ball (a smeared attractor), the map of the simple deterministic system shows a structure, albeit a simple structure.

But can a deterministic system really generate complex and surprising structures, such as the forms we observe in the world and seek to understand and explain through the use of quantitative modeling? Figure 5 shows one of the more famous solutions to a deterministic equation, an image known as the “Lorenz attractor.”



**Figure 5: Lorenz strange attractor<sup>25</sup>**

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25. [http://upload.wikimedia.org/wikipedia/commons/thumb/f/f4/Lorenz\\_attractor.svg/600px-Lorenz\\_attractor.svg.png](http://upload.wikimedia.org/wikipedia/commons/thumb/f/f4/Lorenz_attractor.svg/600px-Lorenz_attractor.svg.png).

Watching the formation of a Lorenz attractor from the time-sequenced solution of a Lorenz equation, we see the structure traced out beginning at the bottom center, curving around to the right and then beginning the tight spiral in the middle of the left side, spiraling outwards for several loops and then shifting to the inside of the right side where it continues to spiral outwards. The shifting from left to right and the spiraling outwards continues indefinitely and aperiodically. As described by noted chaos mathematician Steve Strogatz, “The number of circuits made on either side varies unpredictably from one cycle to the next. In fact, the sequence of the number of circuits has many of the characteristics of a random sequence.”<sup>26</sup> Here we have very complex and surprising behavior, even apparently random behavior, emitting from a simple deterministic equation.

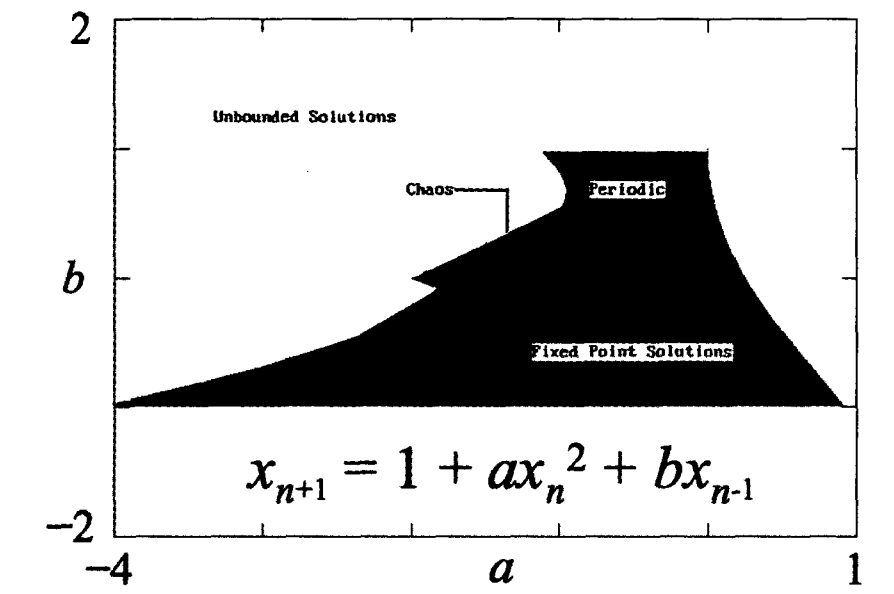
### III. LOW-DIMENSION EXAMPLE

Let us pursue this question of whether a deterministic system can produce complex forms by introducing the element of dimensionality, or degrees of freedom. We can examine a very simple system with only two dimensions in order to survey the capacity of such a system to produce chaotic behavior. Figure 6 is an example known as a “Henon Map.” It displays the solutions for a system with two dimensions,  $a$  and  $b$ .

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26. STROGATZ, *supra* note 2, at 319.

## A 2-D Example (Hénon Map)



**Figure 6: 2-Dimensional Example: Henon Map<sup>27</sup>**

Figure 6 shows four different types of solutions for the deterministic equation: the broad base of the figure represents “fixed point” solutions. These are solutions to the equation that occupy a single point on the plane. Above these are a set of “periodic” solutions. These are solutions that visit two or more steps in order. The vast area around the structure shows the unbounded solutions, those solutions that go off to infinity over an infinite amount of time. Finally, on a sort of “beach” on the North-West edge of the structure, there is a set of chaotic, aperiodic solutions to the equation. The chaotic solutions occupy about 6% of the area of the bounded solutions.

27. See SPROTT, *supra* note 2, at 132.

The solutions to the Henon Map show that a two-dimensional deterministic system can generate a relatively small amount of chaotic solutions. But the social systems, such as legal or financial systems, that we want to model are very high-dimension systems, containing several thousand degrees of freedom. Finally we examine some preliminary research on whether high dimension non-Brownian systems can generate complex forms.

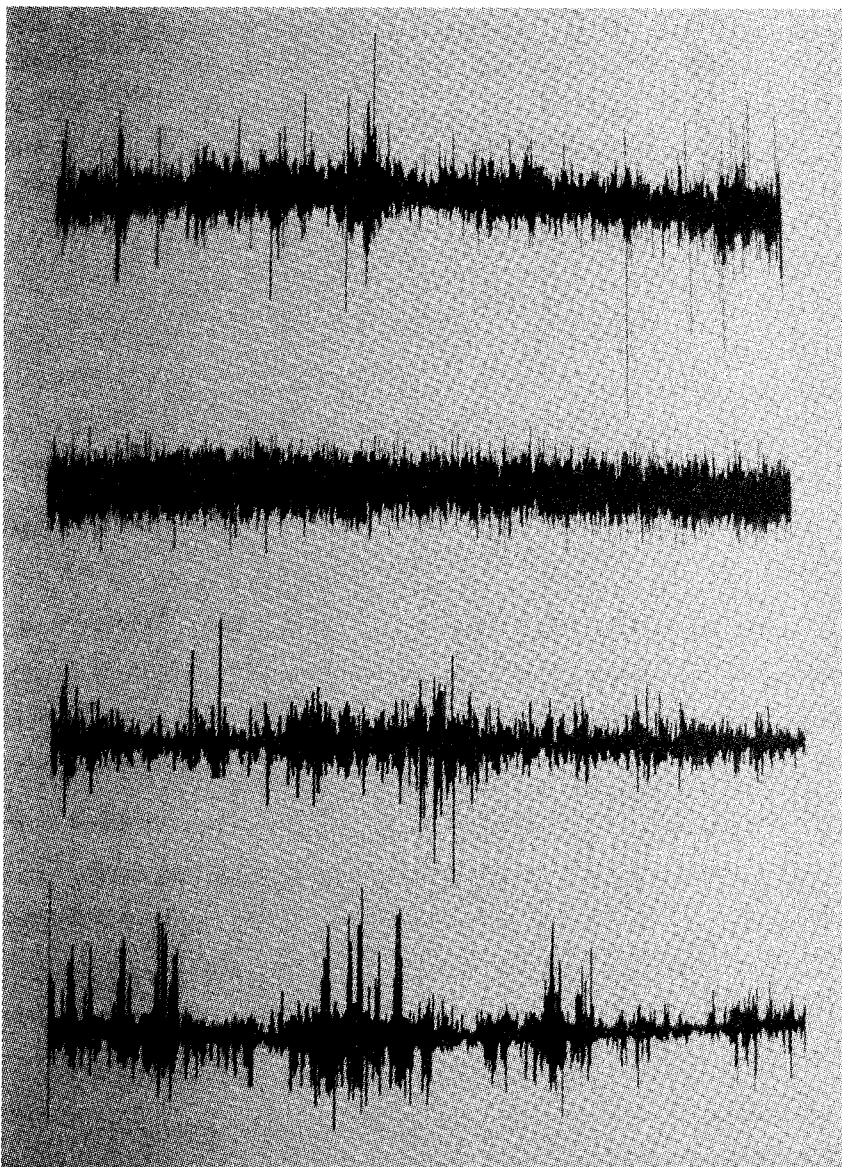
#### IV. HIGH-DIMENSION SYSTEMS

Can deterministic dynamics explain the forms we observe in very high-dimension systems? Clint Sprott, a noted physicist of chaos and complex systems, has shown in a series of papers that certain high-dimension ecological systems can be modeled with simple deterministic equations to produce very complex behavior.<sup>28</sup> Starting with observational data showing the landscape patterns in Southern Wisconsin, Sprott has shown that patterns of similar complexity can be generated by deterministic equations. Sprott's findings show that randomness is not a necessary condition for modeling complex forms in very high dimension natural systems.

One heavily-studied social phenomenon is the stock market. Benoit Mandelbrot, one of the pioneers of complex systems, has recently shown that Brownian randomness (which he calls "mild" randomness) is incapable of generating price movements that resemble the actual price movements of the stock market.

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28. See Julien C. Sprott, *Predator-Prey Dynamics for Rabbits, Trees, and Romance*, FOURTH PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON COMPLEX SYSTEMS at 6 (forthcoming; accessible at <http://sprott.physics.wisc.edu/pubs/paper269.htm>) ("Note that the chaos and spatial structure arise from a purely deterministic model in which the only randomness is in the initial condition"). The deterministic model replicates results from a stochastic model elaborated in earlier papers. See Julien C. Sprott, Janine Bolliger & David J. Mladenoff, *Self-organized Criticality in Forest-landscape Evolution*, 297 PHYSICS LETTERS A 267-71 (2002); Janine Bolliger, Julien C. Sprott & David J. Mladenoff, *Self-organization and Complexity in Historical Landscape Patterns*, 100 OIKOS 541-53 (2003).



**Figure 7: Four data sets from Mandelbrot<sup>29</sup>**

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29. See MANDELBROT, *supra* note 7, at 19 (1: IBM price moves from 1959 to 1996; 2: Model based on Brownian "Random Walk" 3: Dollar/Deustchemarke exchange rate; 4: Mandelbrot model using his fractal geometry).

Figure 7 shows four charts of day-to-day price movements. The first is observational data of the price movements for IBM stock from 1959 to 1996. The second chart shows the results of a model utilizing a Brownian randomness term. The third chart is observational data showing the Dollar/Deustchemarke exchange rate. The final chart shows the results of a model designed by Mandelbrot, using his fractal geometry and an assumption from the Lévy family of probability distributions that Mandelbrot has named “wild” randomness.<sup>30</sup>

At a glance, it is obvious that the “Brownian” model (the 2<sup>nd</sup> chart) looks nothing like the observational data, and that Mandelbrot’s model (the 4<sup>th</sup> chart) does resemble the observational data.<sup>31</sup> Mandelbrot’s showing thus moves our analysis to the next level. Sprott’s work on spatial-temporal landscapes showed that chaos *may* be a viable alternative to the usual Brownian randomness. Mandelbrot shows that, at least for the (extremely important) phenomenon of the financial markets, Brownian randomness cannot generate useful models.<sup>32</sup>

What sort of math can model the financial markets? Mandelbrot’s model relies on a form of randomness he names “wild” (to contrast with the “mild” form of Brownian randomness). We can illustrate the difference between Brownian and “wild” distributions by telling a story of two archers. First, assume an archer of reasonable skill shooting arrows at a target on a wall. Some arrows will hit the target and most will hit near the target. Only a few of the arrows will veer far from the target. So it goes with the construction of a Brownian

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30. MANDELBROT & HUDSON, *supra* note 7, at 19. These charts are collected from earlier work by Mandelbrot. MANDELBROT, *supra* note 7, at 18, 19–23, 183, 184.

31. See also J.C. Sprott, *Competition with evolution in ecology and finance*, 325 PHYSICS LETTERS A 329–333 (2004) (developing a deterministic model that reproduces volatility of stock prices, and showing that a Gaussian model does not reproduce stock movements).

32. For an early law journal critique of the Efficient Capital Markets Hypothesis from the standpoint of complexity science, see Lawrence A. Cunningham, *From Random Walks to Chaotic Crashes: The Linear Genealogy of the Efficient Capital Markets Hypothesis*, 62 GEO. WASH. L. REV. 546 (1994) (critiquing the Efficient Capital Markets Hypothesis from the standpoint of complexity science).



distribution. Most of the data points will cluster around the mean and the variance will be relatively small.

Now imagine an archer of extraordinary strength, who is shooting arrows a mile or more at a target painted on a wall of infinite length. This archer is blindfolded, and is shooting in any direction.<sup>33</sup> Many of the arrows do not even hit the wall. Many hit the wall, but very far from the target. And some arrows hit the wall within a reasonable distance of the target. If the blindfolded archer fires arrows for an infinite period of time, the variance of the arrows around the target will be infinite.

Along with the characteristic of infinite variance, the “wild” distribution also differs from well-behaved Brownian distribution in that the “wild” distribution exhibits some dependence,<sup>34</sup> while Brownian randomness is, by definition, independent point-to-point.

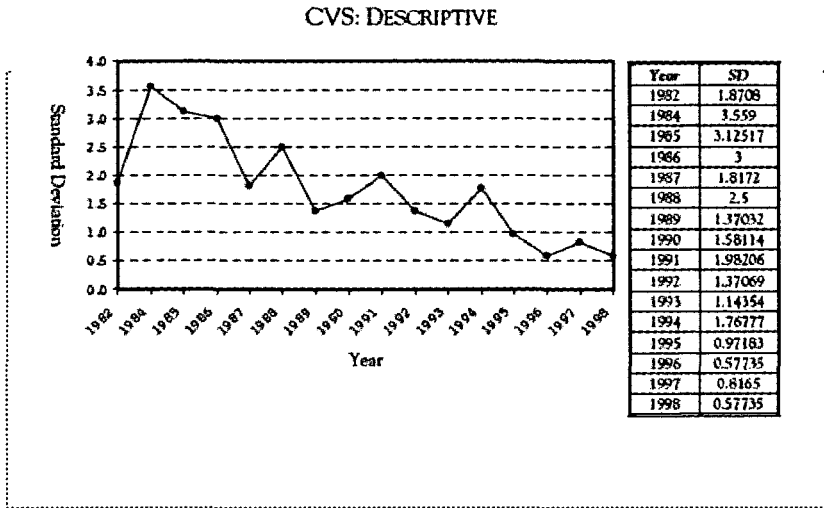
A second example of complexity modeling in high-dimension social systems arises in the area of bankruptcy law. Figure 8 shows a time series of the standard deviation in the “creditors’ valuation standard.”<sup>35</sup>

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33. MANDELBROT & HUDSON, *supra* note 7, at 37–39 (using the blindfolded archer to explain the difference between Gaussian distributions and the work of Augustin-Louis Cauchy).

34. Described by Mandelbrot as “long memory.” Mandelbrot sees clusters of volatility both in observational market data and in his own models, and notes that there can be dependence (i.e. clusters of volatility day-to-day) without correlation (i.e. no predictability as to whether the volatility will trend upwards or downwards). See MANDELBROT & HUDSON, *supra* note 7, at 247–48. Note that what Mandelbrot names “wild randomness” does not qualify for the “point-to-point independence” definition of randomness set out at *supra* Part I, because Mandelbrot’s wild distribution exhibits dependence. By “random,” Mandelbrot seems to mean primarily “unpredictable.” See MANDELBROT, *supra* note 7, at 16 (“The original French phrase ‘un cheval a random’ ... merely served to denote an irregular motion the horseman could not fully predict and control.”).

35. Roughly, how creditors in business cramdown bankruptcy cases would litigate about value. This variable is explained in Bernard Trujillo, *Patterns in a Complex System: An Empirical Study of Valuation in Business Bankruptcy Cases*, 53 UCLA L. REV. 357 (2005). See also Bernard Trujillo, *Self-Organizing Legal Systems: Precedent and Variation in Bankruptcy*, 2004 UTAH L. REV. 483 (2004) (same database); and Bernard Trujillo, *Regulating Bankruptcy Abuse: An Empirical Study of Consumer Exemptions Cases*, 3 J. EMPIRICAL LEGAL STUDIES 561 (2006) (consumer bankruptcy data), arguing that bankruptcy shows a tendency to self organization.



**Figure 8: Creditor Valuation Standard over time<sup>36</sup>**

Figure 8, observational data from 1982 to 1998, displays an obvious decline in the variability of valuation standards by creditors in business cramdown cases. I have previously argued that these data and others may show a tendency of the U.S. Bankruptcy system towards self-organization, in that the dimension of the attractor diminishes over time. Generally speaking, we can say that the 1978 overhaul of the U.S. Bankruptcy laws constituted a “re-set” of the system and established an initial condition of wide variation across the doctrine-space (which variation is shown in the early years of Figure 8). Over time, the size of the attractor settled down to a smaller sector of doctrine-space. From an initial sprawl in the way that litigants talked about valuation (the larger variation early in the data), we see a tighter range of variation later in the data.

36. See Bernard Trujillo, *Self-Organizing Legal Systems: Precedent and Variation in Bankruptcy*, 2004 UTAH L. REV. 558 (2004).

## CONCLUSION

This Article has argued that Brownian randomness may explain much less of social phenomena than is commonly believed and is commonly deployed in modeling. On the other hand, the chaotic determinism of complex systems and wildly misbehaving fractal distributions may have as yet underutilized explanatory power.